This paper presents a new conceptualization of the origins of numerical competence in humans. I first examine the existing claim that infants are innately provided with a system of specifically numerical knowledge, consisting of both cardinal and ordinal concepts. I suggest instead that the observed behaviors require only simple perceptual discriminations based on domain-independent competencies. At most, these involve the formal equivalent of cardinal information. Finally, I present a "non-numerical" account that characterizes infants' competencies with regard to numerosity as emerging primarily from some general characteristics of the human perception and attention system.

In recent years, developmental psychologists have turned with renewed interest to a critical question in their field of inquiry: what are the roots or foundations of our cognitive system? The recent explosion of research on infant competence has had a significant impact on the search for answers to that question. Also important has been the recent renewal of the position that animal models of cognition can provide guidance to understanding infant competence (Gallistel, Brown, Carey, Gelman & Keil, 1991). In this view, humans are perceived as no different from other successful animal species that pass along to their offspring many evolutionary adaptations in the form of innately-specified competencies. Such a view has spawned exciting new research into the competencies of human infants that may be innately specified (Carey & Gelman, 1991). There is also considerable debate about what "innateness" really means, especially in the light of recent findings.
about the extraordinary plasticity of the developing human brain (Elman, Bates, Johnson, Karmiloff-Smith, Parisi & Plunkett, 1996). In particular, a great deal of attention has been paid to infants' ability to reason about events involving inanimate physical objects (Baillargeon, 1993; 1995; Spelke, Breinlinger, Macomber & Jacobson, 1992). A natural consequence of such a program of research is a debate about how initial specifications are transformed into later competence, or how innate competence affects the process of development (Baillargeon. 1995; Carey, 1991; Spelke et al., 1994).

Simon and Halford (1995) also address this debate in a more general fashion as part of an analysis of how one constructs detailed accounts of cognitive change (in their case, in the form of computational models). Along with specifications of the mechanisms of change and the activities the agent must undergo in order to effect change, Simon and Halford point out that a critical element in any developmental account is an accurate specification of what they call the foundational competencies. These are the abilities that a child (or adult) possesses before the cognitive change of interest takes place. In other words, they are the initial conditions or the bootstrapping competencies out of which more sophisticated competencies will be constructed. Should one either underestimate or overestimate that initial set of competencies then the entire account of development will be adversely affected. If too much initial competence is assumed there will be less for the transition mechanisms to do and the tasks required to effect the change will be rather minimal. If too little competence is assumed there will be an overestimate of what transformations will need to occur and of the activities that will have to be engaged in to cause the change to take place.

In this paper I shall present a new position in the current debate about the foundational competencies which infants possess in the domain of numerical competence. Like Zelazo's (1996) analysis of "minimal consciousness" in infancy, my goal is to present the most limited and parsimonious account of numerically-relevant competencies that human infants possess, so that progress towards a comprehensive and accurate account of full number development can be made. My claim will be that the earliest form of numerically-relevant behavior of which infants are capable arises from the deployment of some very general information processing characteristics of the human cognitive architecture, and that these are the origin of our ability to construct and use specifically-numerical knowledge and competencies. Because I am assuming these foundational competencies to be ones that apparently did not evolve specifically for the purpose of number processing, I shall term them "non-numerical" competencies. In the second half of this paper I shall explain this non-numerical view in detail. However, before presenting this conception of the origins of number knowledge it is necessary for me to first address an existing, and rather different, view that has been recently stated by Wynn (1995).

Wynn's (1995) position is that human infants possess "extensive numerical competence" (p. 35). Specifically, she claims that infants "have a means of determining and representing number per se. Furthermore, infants are able to reason
numerically about these representations; they possess procedures for operating over these representations in numerically meaningful ways, and so can appreciate the numerical relationships that hold between numerical quantities. They can thus be said to possess a genuine system of numerical knowledge. These early capacities suggest the existence of an unlearned basic core of numerical competence” (p. 35). Such a view implies that infants possess the ability to represent and use both cardinal and ordinal concepts of number.

I shall first clarify the distinction between cardinal and ordinal concepts of number and the minimal competencies required in order to operate with them. Then I will address some of Wynn’s claims of extensive infant numerical competence. Finally, I will present my non-numerical processing account of infant numerical competence.

CARDINAL AND ORDINAL CONCEPTS

My claim is that the apparently-numerical behavior in which infants have been seen to engage actually requires no specifically-numerical knowledge. Instead, all the results produced so far require only the detection of a same/not same discrimination. Although it is certainly true that, in making simple discriminations, infants’ behavior is consistent with arithmetical operations based on cardinal representations of quantity,¹ this does not mean that they possess the conceptual competence of number and arithmetic.² Their behavior can be non-numerical because such discriminations can be made with competencies that have no direct connection to the conceptual domain of number.

Dantzig (1954) begins his seminal book on number with the following claim about the basis of numerical competence in human civilization:

Man, even in the lower stages of development, possesses a faculty which, for want of a better name, I shall call Number Sense. This faculty permits him to recognize that something has changed in a small collection when, without his direct knowledge, an object has been removed or added to the collection.

Number sense should not be confused with counting, which is probably of a much later vintage, and involves, as we shall see, a rather intricate mental process (1954, p. 1) [Original Italic].

1I thank Randy Gallistel (personal communication) for making this point apparent to me.
2Although logicians refer to determining that one entity is not the same one as another entity as establishing “numerical identity,” it does not necessarily follow that the processing required to do this is inherently “numerical.” Indeed, since the perceptual system is undertaking the task of determining whether one perceived entity exists independently of another, it might be just as appropriate to refer to such individuation as establishing “existence identity” of the individuals involved. However, given that the separate existence of such entities can be established, these competencies can then be used to make simple, quantitatively-relevant judgments such as “more” or “less” in the way described by Dantzig (1954). Because such judgments can be said to be numerical in nature, I claim that primitive, perceptual capabilities create the conditions for carrying out these simplest of numerical operations. However, since the primary purpose of individuation processes seems not to be directly related to the domain of number, I think the best way to characterize the competence of infants is “non-numerical.”
This is an important claim because the kind of competence that Dantzig calls number sense is exactly that which is required of an infant in the kind of studies that Wynn (1995) interprets as providing evidence for extensive numerical competence. However, unlike Wynn, Dantzig is stating an extreme limitation to this competence. He is claiming that number sense supports only simple discriminations between collections (perceived or remembered) and only in the realm of small quantities. In this view, only representations functionally equivalent to cardinal concepts are required for such competence. Dantzig explicitly distinguishes this ability from counting, which can be used to quantify collections of any size, and more importantly, which relies on the ordinal concept of numerical relations between collections.

The cardinal representations of which Dantzig spoke allow collections of discrete objects and events to be discriminated one from another on the basis of one to one correspondence. Such a process requires two collections of objects in order to determine relative quantity. As Dantzig states, relative quantity can be determined through discrimination which “consists in assigning to every object of one collection an object of the other collection, the process being continued until one collection, or both, is exhausted” (p. 7). Thus a person unable to count and with no appreciation of the ordinal relation between the quantities could use this method to determine, for example, the relative quantity of males and females in a room. Everyone is simply asked to stand and then one member of each gender sits down. If a member of either group is left standing alone then that group is the more numerous.

To determine absolute quantity there must be one collection of objects to enumerate and one collection of tokens. These latter are arbitrary entities such as a mark made on a page for each object encountered, fingers held up in the same way, or symbols which can be physically present or mentally represented. Here Dantzig states that the “transition from relative number to absolute is not difficult. It is necessary only to create model collections, each typifying a possible collection” (p. 7). In other words a mapping can be established between an arbitrary symbol or set of symbols and the size of the collection that it represents. For most of us the string “III” stands as a model collection for any real collection of three objects, as does the single symbol “3.” All that is necessary is that we learn the unidirectional (in this case) mapping involved. Every time we see “III” we can envisage that many cows, cookies or cars. Nothing in this scheme requires that we understand the relation of the collection denoted by “III” to that denoted by “I I” for example. For us, “III” is just a simple model for any three things we wish to refer to.

Operating with ordinal or relational representations of quantity requires a far more elaborate set of competencies. As Dantzig states:

The cardinal number rests on the principle of correspondence: it implies no counting. To create a counting process it is not enough to have a motley array of models, comprehensive though this latter may be. We must devise a number system: our set of models must be arranged in an ordered sequence, a sequence which progresses in the sense of growing magni-
tude, the natural sequence: one, two, three... Once this system is created, counting a collection means assigning to every number a term in the natural sequence in ordered succession until the collection is exhausted. The term of the natural sequence assigned to the last member of the collection is called the ordinal number of the collection (1954, p.8) [Original Italics].

Unfortunately there is some confusion in this terminology. Just such a counting process is described by Gelman and Gallistel (1978) and yet they term the output of counting the "cardinal" numerical value of the collection, and name the counting principle which states that rule the "cardinal principle." In what follows I shall retain Dantzig's more widely accepted terms and refer to relational numerical concepts as ordinal. In any event, the above definition of ordinal representations stands as a strong prediction for infant competence. If Wynn is correct in her claims about infants' understanding of number, there should be evidence that they appreciate the ordered sequence or relationships between different quantities rather than that two collections are merely "equal" or "not equal" to one another. The question then becomes whether there is any evidence of such understanding and, if not, whether anything more than cardinal representations are required to explain infants' behavior on the tasks in question. So as not to set up expectations that cannot be met I shall state that, at present, I know of no evidence that infants appreciate ordinal relations between quantities. Despite Wynn's claims, no-one has yet shown that infants can determine the direction or magnitude of change in collections of objects whose numerosity has been manipulated.

WYNN'S CLAIMS

Among the claims that Wynn makes for infants' numerical knowledge are three that I will address in detail here because of their centrality to the debate over the nature of infants' numerical competence. The first claim is that infants understand the true numerical value of a collection of objects and make distinctions between sets based not on perceptual details but on numerosity per se. The second is that their enumerative ability is general because it can be applied to collections of all kinds of entities. The third claim is that infants understand the ordinal relations between different collections that they can enumerate. The first two claims imply that infants are not simply sensitive to perceptual discriminations based on the existence of distinct items that make up different collections. Rather they state that infants possess some kind of conceptual knowledge about the numerosity of the entities involved. While I will not claim that such an interpretation is impossible, I shall show that simple cardinal discriminations are all that is needed in order to generate the behavior that infants exhibit. The third claim is far stronger. It suggests that, not only do infants have a conceptual representation of quantities, but that they understand the ordinal relations between them. In other words, when the size of a collection is altered, infants understand in which direction the change has occurred, and by implication, to what extent, and how a collection of one quantity can be transformed into a collection of another quantity. Apart from the fact that
none of the existing findings directly demonstrate such understanding, I shall claim that the behaviors upon which Wynn’s interpretation is based do not even require this kind of ordinal knowledge. Again, all that is required is a cardinal representation and some non-numerical knowledge about the behavior of the objects involved.

**Are Infants Sensitive to Numerosity Per Se?**

Wynn’s (1995) claim of infants’ sensitivity to numerosity seems to be best characterized by the following statement. When referring to infants’ ability to discriminate between collections of almost any kind of individual she writes “[t]his generality, in turn, indicates that the enumeration mechanism cannot be operating over physical or perceptual properties of the entities, recognizing certain perceptual patterns, and the like: it must be determining number of entities per se” (p. 46). Wynn bases her claim on evidence that newborns and older infants are able to discriminate small collections of items from one another. This has been demonstrated in a number of studies that used a habituation design to demonstrate such discrimination between collections containing up to four items but not more (Antell & Keating, 1983; Starkey & Cooper, 1980; Strauss & Curtis, 1981). As Wynn points out, this is roughly the same limit as the ability demonstrated in older children and adults known as subitizing; a rapid and accurate enumeration profile where up to four items can be enumerated at around 50 milliseconds per item. Subitizing contrasts with counting, which is employed to enumerate larger collections at a rate of around 300 milliseconds per item in adults and where errors are made at a rate of 20% or more (Atkinson, Campbell & Marcus, 1976; Trick & Pylyshyn, 1993). Wynn’s point is the following. If subitizing is merely pattern recognition then it requires no numerical knowledge. Therefore, discrediting the view that infants’ competence on these tasks is based purely on pattern recognition would imply, instead, that babies’ responses are based on true numerical knowledge. To this end, she cites the findings of Starkey, Spelke and Gelman (1990) showing that infants can discriminate both heterogeneous sets of objects from one another, and cross-modally presented sets (objects and sounds) from one another. Since these discriminations are not made on the basis of the perceptual details of the objects being presented, Wynn claims that infants’ ability to execute them discredits the view that infants are employing subitizing, which she equates to using pattern recognition.

Therefore, Wynn’s position is that infants’ representations of collections of objects are fully-developed cardinal and ordinal number concepts. It seems, consistent with statements made elsewhere (Wynn, 1992), that Wynn is suggesting that infants’ ability to determine number per se means that they are able to fully understand the semantics of concepts like “two” or “three” both in themselves and in how they stand in relation to other concepts like “one” or “four.” Put another way, infants are not simply recognizing that “two” is different from “three” in the
same way as "two" is different from "blue." Rather, the claim is that infants know, as we do, the essentially numerical ways in which "two" is different from "three."

However, Wynn bases her argument against subitizing on a theory which has been called into question by recent findings, and whose thrust does not appear to support the point she wishes to make. Mandler and Shebo's (1982) theory of subitizing is that it is the result of recognizing canonical patterns that are learned during the lifespan. Because one item is always a singleton, two always form a line and three (almost always) form a triangle, Mandler and Shebo claim that repeated exposures to these patterns leads eventually to instantaneous recognition of them as model collections for the quantities one through three. However, there are numerous problems with this interpretation. The theory predicts no slope in subitizing reaction time when almost every study reports one of at least 20 milliseconds per item, it has no account for the oft-demonstrated ability to subitize four objects, and much evidence exists to show that collections of stimuli which have no such patterned characteristics can also be discriminated. One such study (van Loosbroek & Smitsman, 1990) demonstrates this last point very well by using sets of complex, moving items that even partially occlude one another. Here the subjects were not adults but infants. Mandler and Shebo's account claims that the whole reason that canonical patterns allow subitizing to occur is that they hold constant the patterns for the first three collections, thereby allowing learning of their reference as model collections during childhood. Because subitizing-like performance has been demonstrated in newborns (Antell & Keating, 1983) it is clear that such a learning account is not necessary.

Wynn's claim is that, because infants are not influenced by the perceptual nature of entities during enumeration, their ability must instead be based on a conceptual appreciation of numerosity. However, such a claim cannot be used to discount explanations of of subitizing based on pattern matching (such as Mandler & Shebo's) as Wynn wishes to do. A theory based on canonical patterns turns not on the appearance of the objects being presented but on their configural characteristics. In other words, for Mandler and Shebo it makes no difference whether the stimuli are a homogeneous or heterogeneous group of objects. Their theory simply states that fewer than four objects will form a canonical configuration whose quantity will thus be immediately apparent due to its lifelong familiarity to the perceiver. Therefore, Wynn's claim that infants' abstraction of the perceptual details of the stimuli means that the resulting representation is explicitly numerical and not perceptual in nature is not logically necessary. It is merely a demonstration that infants are able to abstract over the perceptual details of individual items in the service of carrying out a discrimination between two collections based on one to one correspondence. This is a point to which I shall return later.

I thank David Klahr for this example.
Are Infants' Enumeration Skills General?

Wynn (1995) claims even broader enumeration competence is present in infants by citing evidence from her own studies that 6-month-olds can discriminate sequences of jumps executed by a puppet. Infants witnessed, on a stage, a puppet that made a series of jumps; either two or three times. Jumps were varied in tempo and duration of sequence to rule out responses based solely on the temporal dimension. Looking time was measured at the end of each jump sequence and habituation was observed when a sequence of the same number of jumps was repeated. The baby then saw a sequence of a different number of jumps and the results showed dishabituation to the novel sequence of jumps. The same was true, but with a smaller difference in looking time, when the puppet continued to move slightly between each individual jump.

From this, Wynn claims that infants' enumeration skills are general because they can discriminate both between sets of discrete entities and between sequences of events. The events used in the puppet-jumps task are neither as discrete as individual objects or sounds (especially when the puppet continues to move slightly between jumps) nor are they all present at the same time, as are collections of objects. Wynn (1995) claims that for infants to distinguish between different sequences of up to three puppet jumps they "must have been sensitive to the number of jumps in each sequence." However, the claim that parsing a continuous or semi-continuous sequence (of very limited length) into sequence of discrete units indicates a general enumerative competence seems unwarranted. The ability to perceive a temporally extended event, such as singing a song or playing a game, as unitary does not logically imply that such an ability represents a general enumerative competence. Instead, only the following abilities are required. The infant must be able to represent superficially similar objects or events as equivalent units. The appropriate dimension along which they would be similar would seem to be "existence" in that each discrete entity exists as a unit in a collection. The infant must also be able to parse temporal streams into discrete unitary categories; for example where each unit is "a jump." Finally the infant must be able to compare one collection to another as in Dantzig's definition of cardinal discrimination.

Do Infants Understand Ordinal Relations?

This would seem to be the critical test for Wynn's claims. If infants do indeed possess an extensive system of numerical knowledge and can understand arithmetic transformations, then they must be able to detect and understand ordinal relations between quantities. Wynn's (1995) claim that infants "possess an appreciation of the numerical relationships that hold between different small numbers" (p. 41) is based on findings that infants look longer at impossible outcomes involving the appearance and disappearance of objects hidden behind a screen (Wynn, 1992). The behavior of the infants engaging in such a task is not in question. Indeed the results have been replicated by other researchers including my colleagues and me.
Origins of Number Knowledge

Figure 1. Schematic of Processing Required for One to One Discrimination.

(Simon, Hespos & Rochat, 1995). What is open to dispute is the interpretation of what such behavior means in terms of infants' understanding of ordinal relations and simple arithmetic.

In Wynn’s original study, 5-month-old infants saw a Mickey Mouse doll placed on a stage that was then occluded by a screen. Next, another identical doll was
moved into view, placed behind the screen and the hand that carried it emerged empty. This is what Wynn called the “addition” condition. There were two types of outcomes. In the “possible” situation the screen was lowered and the child saw a Mickey Mouse doll and a Mickey Mouse doll next to each other. In the “impossible” situation the screen was lowered and all the child saw was a lone Mickey Mouse doll. (There was also a “subtraction” condition where a doll was seen to be removed from a hidden pair but then was still present when the screen was lowered). As in most “violation of expectation” experiments the dependent variable was relative looking time to the different outcomes. Babies looked reliably longer at the “impossible” outcome, a result usually interpreted to mean that their expectation was that such an eventuality should not occur. As Wynn (1995) herself states “these results do not show [the infants] are anticipating the precise nature of the change” (p. 43). Therefore, she reports a further experiment where the original “addition” condition was carried out but instead of the impossible situation containing one doll too few, it now contained one doll too many. (In arithmetical terms, one plus one now equaled three instead of one). Wynn (1995) claims that the infants’ longer looking time to the new impossible situation is evidence that they “are computing the precise result of the operation; they know not only that the display of objects should change, but exactly what the final outcome should be” (p. 43).

The interpretation from the original experiment that longer looking time indicated an expectation-violation for the “impossible outcomes” will not be disputed. However, what is at issue is on what basis did the infants decide that the result was impossible. Was it because they understood ordinal concepts and knew that “one plus one equals two,” or was it merely because an object that previously existed behind the screen unexpectedly disappeared? There is no evidence from these studies that infants appreciate the ordinal relation between the observed and outcome states. They are required only to make a “same/different” judgment which can be done using one to one correspondence as in Dantzig’s description of the discrimination of relative cardinal number. In this case, one collection of objects is a set of mental “unit” tokens held in memory which represent the items that were placed behind the screen, while the other set is the newly revealed objects on the stage. Merely by aligning one object of each set until either or both are exhausted the child can determine a “same/different” judgment. Just such a process is depicted in Figure 1.

In the first frame, the infant is presented with a doll on the stage and creates an abstract representation of that object in the form of a mental token (the precise nature of this type of representation will be dealt with in detail later). In the second frame the screen is raised, occluding the infant’s view of the object but the mental token representing it remains in place since the infant assumes that the object persists behind the screen. In frame three we see another doll being added behind the screen, and the infant accordingly creates another mental token. In the fourth frame, unbeknownst to the infant, one of the dolls is removed from the stage. Because the infant does not witness this, no change is made to its mental represen-
tation of the objects behind the screen. In the fifth frame the screen is lowered. The infant now forms a mental representation of the new scene by creating a single mental token for the lone doll on the stage. The previous tokens, now enclosed in a rectangle, denote the memory of the previous state. In the sixth frame the infant initiates the process of one to one comparison by pairing off an object from each set (the tokens for the visible and the remembered objects). As we can see, the first objects from each set are successfully paired, denoting equality of the representations so far. In the final frame the tokens representing the visible objects have been exhausted while at least one token remains in the remembered set. This means that a remembered object that was expected to be present is not present. Hence the infant has established inequality between the remembered and visible objects and experiences a violated expectation of equality.

Similarly, there seems to be no basis for Wynn’s interpretation that results of her second experiment showed that infants computed the precise result of the operation. The new impossible situation is no different in terms of its impossibility than was the original one. Both can be detected using the same one to one discrimination process illustrated in Figure 1. All an infant has to do is to once again align a mental unit token with a visible doll and it will find that the collections do not match. No appreciation of the direction of change is required. This time it is the set of tokens representing the remembered items that is exhausted before those representing the visible ones.

Summary

Thus far I have evaluated one view of the origins of number knowledge; Wynn’s claim that infants possess a system of numerical knowledge which involves the innate specification of ordinal as well as cardinal concepts, thereby endowing the infant with an understanding of simple arithmetic. I have not argued that this eventuality is impossible. Rather I have presented the view that the evidence Wynn cites in favor of her interpretation of infants’ behavior can be accounted for by the existence and use of the much more primitive representations that are functionally equivalent to cardinal concepts of quantity. Next I will present in more detail an alternative account of infants’ competence in these tasks.

A “NON-NUMERICAL” ACCOUNT OF INFANTS’ BEHAVIOR ON QUANTIFICATION TASKS

If infants do not start life with a system of numerical knowledge involving ordinal concepts, how then do they produce the behavior in the experiments described in the studies discussed earlier? My answer to that question starts from the assumption that the human cognitive architecture with which infants are provided, is constructed in such a way as to enable sophisticated perception, reasoning and learning abilities immediately after birth. This is a very general set of competencies designed to enable the child to interact with an unpredictable and changing
world. One implication of this view is that certain functions will be co-opted into tasks that they can successfully accomplish without having been specially designed for such a purpose. This claim is simply the assumption that, when the human cognitive system finds itself faced with a novel task, resources or competencies that are appropriate for responding to that task will be called into service, regardless of the original adaptive response that led to their development. It is my contention that what has been described as numerical processing in the infant is an example of just such a co-option. Thus, the behavior that has been described as infant numerical competence is instead merely the result of a general purpose system that is not specifically numerical in nature being exercised on tasks that it is presented with; such as making same/different discriminations between sets of entities. In other words, although such a discrimination can be correctly described as numerical in nature (albeit rather primitive), it might be that infants are not really engaging in the goal of making numerical judgments but are simply examining their world for any changes that might convey some new information.

It is for this reason that I have labeled my explanation of the behavior of infants in these tasks a "non-numerical account." Before describing them in detail, I will briefly review the set of competencies that I claim make up the most parsimonious description of the initial conditions for an infant to produce the behavior we have observed. These conditions create a set of requirements which, if they can be satisfied, will constitute an alternative conception of the origins of numerical competence. This minimal set, along with some comments and representative references, is presented in Table 1.

### Table 1. Minimal Competencies Required For "Non-Numerical" Account.

<table>
<thead>
<tr>
<th>Competence</th>
<th>Comments</th>
<th>Representative References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Memory &amp; Comparison of Perceived</td>
<td>Basic requirements for habituation phenomenon in human</td>
<td>Cohen (1991)</td>
</tr>
<tr>
<td>Entities</td>
<td>infants.</td>
<td></td>
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<tr>
<td>Limited Individuation and</td>
<td>Individuation of small sets of entities. Supports</td>
<td>Antell &amp; Keating (1983)</td>
</tr>
<tr>
<td>Discrimination</td>
<td>cardinal discriminations.</td>
<td>van Loosbroek &amp; Smitsman (1990)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Simon &amp; Vatshnavi (1996)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Trick &amp; Pylyshyn (1994)</td>
</tr>
<tr>
<td></td>
<td>Represent distinct entities as equivalent &quot;units&quot;.</td>
<td>Narter &amp; Rosser (1996)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Xu &amp; Carey (1996)</td>
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<tr>
<td></td>
<td></td>
<td>Kahneman et al. (1992)</td>
</tr>
<tr>
<td>Physical Reasoning</td>
<td>Interpret object movement and existence in line with physical laws.</td>
<td></td>
</tr>
</tbody>
</table>

*Note: * Infant studies, Adult studies
All of the studies that appear to demonstrate infant numerical competence used either habituation or violation-of-expectation paradigms which means that either familiarization and test collections or original and transformed collections were never visible at the same time. Therefore, infants would need the ability to remember and compare items from a previously viewed collection to those in a currently visible collection. This kind of competence is a basic requisite for the habituation paradigm to work and so has been demonstrated countless times in infant research (Cohen, 1991). Therefore, we can assume that the general case of an ability to compare remembered and perceived items is present in infants at 5 months of age. As such this will be taken as a given and it will not be dealt with further in this paper.

In the studies reviewed, infants needed to be able to make discriminations between collections of up to four objects and I have claimed that this ability is in some way related to the phenomenon of subitizing, that has been demonstrated in infants. Therefore, 5-month-old infants should possess competencies that enable them to produce discriminations that resemble the subitizing phenomenon. Another requirement is the ability to form representations that generalize over some or all of the perceptual details of the actual items involved. This would either require the formation of simple perceptual categories where a heterogeneous set of entities is treated as similar by virtue of sharing a value on some dimension. One such dimension could be “existence” where all perceived objects would have a positive value. Thus a heterogeneous collection of visible objects would be represented as all belonging to the same category. Finally, infants would need sufficient physical reasoning abilities so that the disappearance of an occluded object (or the conversion from a positive to a negative value on the existence dimension) would be noticed and result in a violated expectation. These competencies, along with the ability to execute a one to one mapping between two collections of objects (such as that in Figure 1) in order to carry out quantitative discrimination, are the requirements for my non-numerical account of the foundations of numerical competence.

The necessary implication of my view, then, is that the system of numerical knowledge that Wynn ascribes to infants is, in fact, not present. In other words, the conceptual system of extensive numerical knowledge claimed by Wynn, and the numerically-relevant competencies of the infant I will specify are incommensurable with one another—that some large scale conceptual change process is required between infancy and early childhood. There are already some theorists who have claimed this is so and have laid out possible accounts of such change both in psychological and philosophical terms (Cooper, 1984; Kitcher 1983). Now, having established the outline of the alternative developmental account, let us examine the set of foundational competencies that I am claiming for the basis for infants’ behavior in the tasks described above.

**Limited Individuation Capacity**

The first requirement was for a competence similar to what has been called subitizing in older children and adults. While subitizing is still not completely under-
stood, there is now converging evidence about its information processing basis. Among others, Simon and Vaishnavi (1996) and Trick and Pylyshyn (1993, 1994) have recently demonstrated that subitizing is primarily the result of an individuation process that takes place in early preattentive processing. Trick and Pylyshyn showed that arrays can be subitized when the target items "pop-out" from the background (such as target O's & distractor X's) and are assumed to be individuated and identified in parallel, while arrays where target identification/individuation requires serial search (e.g., O's in fields of Q's) cannot. Thus we can separate the phenomenon into two phases. The first is merely individuating the stimulus objects, or separating them from one another. This is clearly a process quite independent of any numerical knowledge; it simply establishes the existence of discrete entities. The second phase (for tasks involving children or adults) involves saying how many objects are present, and this clearly does involve numerical knowledge.

For the infant, the second phase would involve carrying out a one to one comparison, either to another visible display or a remembered display, in order determine equality or inequality and thus produce either habituation or dishabituation.

The typical subitizing data are derived from experiments inappropriate for studying infants. So, while I do not wish to claim that infants are actually "subitizing," there is evidence that they do possess the same limited individuation capacity that subitizing experiments reveal later in life. As a result we have a strong clue about one of the foundations of infant numerical competence. If infants share the same early attentional processes that appear to support such tasks, then it is reasonable to assume that they are individuating objects in small collections the same way that older children and adults do. Recent evidence shows that this is indeed the case. Colombo, Ryther, Frick and Gifford (1995) have demonstrated that by 4 months of age infants show the same target/distractor detection profile as adults when the task is converted to a habituation format. Also, Johnson (1994) has demonstrated that at around the same age, infants have developed most of the same covert attentional processes that are present in adults, and which are implicated in the most recent theories of subitizing.

Evidence for this individuation mechanism in infants comes from studies using habituation designs, where it has been shown that babies can discriminate between collections of up to four moving or stationary objects but they can not do so for larger collections (Antell & Keating, 1983; Strauss & Curtis, 1981; van Loosbroek & Smitsman, 1990). These results show the familiar profile of easily represented (and thus discriminable) collections in the subitizing range with failure beyond that point. This failure is almost certainly attributable to infants' inability to count; one of the processes assumed to be responsible for the steeper post-subitizing slopes in older subjects.

Abstract Representation

The convergence between theories of subitizing and visual attention is further evident in the representational constructs that have been proposed for the individua-
tion process, such as FINSTs (Trick & Pylyshyn, 1993), and “object files” (Kahneman, Triesman & Gibbs, 1992). These embody the idea that object recognition is initially carried out on the basis of abstract representational tokens that, at first, carry no information about the identity or perceptual characteristics of the object. These details are filled in by later attentional processes. For example, an object file can be thought of as an episodically defined place holder (or file) into which identity information about a “thing” in the visual field can be added as it is recognized and integrated. This accords with the general notion that the attention system has “what” and “where” subsystems with the “where system” being at least the temporally prior one. In other words, we see our world as composed first of “things” and, only later, by attending to those things in the world do we determine exactly which thing it is. One can think of the process rather like our ability to recognize a “person” walking down the street towards us long before we are surprised to find out that it is actually a specific instance—a person that we know.

Thus, if infants’ comparison of collections of objects is carried out via one to one correspondence based on a subitizing-like process, it appears to take place based on representations that are initially processed as units free of their identity information (Trick & Pylyshyn, 1993). In other words, the process of determining relative quantity may be thought of as one requiring comparison only of spatial (where) information and not identity (what) information. Two recent studies provide evidence consistent with this view. Narter and Rosser (1996) presented 9-month-olds with a single object (a small troll) on a stage, briefly occluded it with a screen and then lowered the screen to reveal the stage again. The object either appeared in a new location, was replaced in the original location by one of a different size (large troll), was replaced in the original location by a same sized but novel object (small bear), or was replaced by a novel and different-sized object (large bear) in a new location. Looking times to the post-occlusion events were used as the dependent measure. Results indicated that infants were only sensitive to changes in size and location (spatial information) and did not look longer at outcomes where identity information changed.

Xu and Carey (1996) have also recently shown that infants can so prioritize spatiotemporal (where) information over identity (what) information that they apparently ignore gross changes of identity in objects that are occluded during some part of a continual movement. Thus, so long as evidently different objects share the same spatiotemporal path, infants at 10 months of age will not respond to them in the same way that an older child or adult will. Xu and Carey describe a number of experiments but the details of one in particular are of most interest here. In the “property/kind” condition of their Experiment 2, infants saw two very different objects (such as a truck and an elephant) appear from opposite ends of a single screen and then move back behind the screen in such a way as to suggest a single path of motion with the object changing identity at each end. In the “spatiotemporal” or control condition, the only difference was that the infants were first shown the two objects together, one being placed to the right and one to the left of the
screen. Then the same procedure as in the previous condition was carried out. The dependent variable was the difference between looking time at the end of the trial, measured when the screen was removed and either one or both of the objects was present, and baseline measures. In both cases the “unexpected outcome” was the case where only one of the objects was visible. In the baseline conditions of all the experiments reported, infants showed a preference for looking at two-object displays. Therefore, significant changes in looking time were defined not as more time spent looking at the unexpected outcome (in absolute terms) but in overcoming the preference for two object displays (i.e., in relative terms, longer looking at the single object “unexpected” displays).

The preference was only overcome in the “spatiotemporal” condition, where it was clear (on the basis of spatial information alone) that more than a single object was partaking in the movements behind the screen. However, in the “property/kind” condition, where no such definitive spatial information was first provided, the unexpected outcome did not increase looking time relative to a single object in the test phase. In other words, without clear evidence that more than one object is behind the screen, coding the visible entities in spatial terms alone makes the property/kind condition an indeterminate one. It might or might not be the same object that is appearing at each end of the screen and so the single object outcome is not necessarily “unexpected”. This pattern of results was replicated in further experiments reported by Xu and Carey, even under full habituation conditions where the infants became very familiar with the different objects before they were moved behind the screen.

Of course, apart from the fact that this result is consistent with the general characteristics of the attention system, it makes sense that young infants would prioritize spatiotemporal over identity information. This is because, during development of the acuity of the visual system in infancy, object detection using cues like motion would prove far more reliable than those based on detailed surface characteristics (Kellman & Spelke, 1983). Xu and Carey refer to this spatiotemporal prioritization as “object first” encoding, intending to indicate that, in these kinds of tasks, infants first represent entities as “objects” before they code them as some specific object. In a replication of Wynn’s (1992) original study, Simon et al. (1995) also found evidence that 5-month-old infants apparently ignored the property information of objects whose existence behind a screen they were required to represent.

The above experiments all employ variations of the habituation and expectation-violation paradigms that have been so central to recent advances in the understanding of infant competence. While little argument exists over the interpretation that extended looking time in the face of an “unexpected” or “impossible” outcome indicates a violation of the infant’s expectations, much more caution must be exercised in the inverse case. A lack of looking time difference, such as in Xu and Carey’s “property/kind” condition, does not necessarily indicate that the infants comprehend the outcome and find it consistent with their expectations. Also,
unexpected outcomes in the spatiotemporal condition do not produce increased looking times with respect to baseline but instead remove a two-object preference (or in operational terms, reduce looking time to the "expected outcome"). Therefore, it is probably unwise to interpret Xu and Carey's results as strong evidence for a particular type of representation that is being used for reasoning about objects. However, they do suggest that infants below 12 months of age either do not or cannot employ identity information in the service of reasoning about the behavior of objects in the kinds of tasks described above.

Other competencies, such as the ability of infants to determine correspondences between collections encoded from different modalities should also submit to explanation by my non-numerical account. For example, Starkey et al. (1990) demonstrated 6- to 8-month-old infants' ability to match up to three auditory signals (such as drumbeats) with the corresponding number of visually presented objects. As explained earlier, the process of one-to-one correspondence requires only the pairing of randomly selected items from each of two collections. There is no doubt that objects presented together to the visual modality can be stored by an infant as a set of memory units. Amongst others, Wynn's puppet-jump studies demonstrate that infants can similarly represent and compare sets of rather less discrete, temporally-separated events. Presumably drumbeats would be remembered in the same way. As Wertheimer (1971) has shown, from birth babies are able to compare inputs from different modalities such as sounds and the location in space of their sources. Thus, there seems to be no reason why memory units cannot be used in the one-to-one comparison process described by Dantzig, despite differences in the modality that led to their creation.

In this section I have argued that one to one correspondence is probably carried out on representations of objects without reference to identity information. However, I do not intend to imply that infants do not or cannot encode and use identity information in detailed and sophisticated ways. However, it is important to make clear that I am using identity here in the sense of sameness based on properties (i.e., the "same kind" of thing) as opposed to that based on number (i.e. the "same number" of things).

Among others, Baillargeon and her colleagues have demonstrated both the use of and developmental progression in the processing of property information in infants. Baillargeon (1993) showed how the degree of similarity between a reference box and one hidden behind a rotating screen affected 4.5- to 6.5-month-old infants' ability to use the reference standard to reason about how far the screen could rotate before coming into contact with the occluded object. Baillargeon and Graber (1987) demonstrated that infants as young as 3.5 months represent at least the height of a toy rabbit as it moves behind a screen with a window cut out of it. Infants exhibit expectation-violations when a rabbit tall enough to be seen in the window passes behind the screen without being visible in the window. However, no such response is seen to the invisible passage of a rabbit too short to be seen in the window. Clearly, infants must be encoding properties of the objects in question
in order to support these different reactions. Why then, if infants can represent and use some kinds of identity information do I claim that they are not doing so in the tasks we are concerned with here? One reason is that, even if encoded, this factual knowledge about the objects may not be informative. Thus, while property information enables infants to make judgments about the kinds of entities they encounter, knowing that a rabbit seen at Time 2 is of the same type as one seen at Time 1 does not tell the observer how many such individuals are present.

Another reason is that encoded factual knowledge may not be required for the procedural task of establishing correspondence between the previously hidden and now-present arrays. Well-established approaches to cognitive simulation like production systems (e.g., ACT-R, see Anderson, 1993) and Soar (Newell, 1990), have shown that, when solving a problem, not all aspects of an encoding are used in processing a given task. To produce the correct behavior, cognitive models use variables in their rules so that only some aspects of the representations of entities are tested. In this way, certain details of those entities can be treated as critical conditions for the application of procedural knowledge while other aspects have no effect on behavior. Because the behavior of production systems (that best embody this distinction) is contingent on the goal of the activity, different aspects of the representation can be discounted, and become coded as variables, depending on what features are relevant to the task.

Thus, in the context of tracking a toy rabbit behind an odd-shaped screen, the infant will need to test certain aspects of the rabbit’s appearance in order to reason appropriately. The infant’s reasoning maybe best characterized by a rule of the form “IF the task is to predict the appearance of the moving object, and the object is moving in a path that takes it behind the screen, and the object is a rabbit, and the rabbit is tall enough to appear in the window in the screen, THEN expect the rabbit to appear in the window.” However, a rule for the type of task used to assess infant numerical competence might read “IF the task is to predict the existence of an object placed behind the screen, THEN expect the object placed there to persist when the screen is removed.” The critical point here is that there is no need for the infant to test aspects of the object’s appearance because property identity is not relevant (in most cases) to the persistence expectations for that object. Of course, it is an empirical question to determine exactly what aspects of objects and entities infants both encode and employ in reasoning, but it is not controversial to claim that problem solvers of any sort represent and test different aspects of memory representations depending on the task at hand.

Finally, some form of abstraction over perceptual inputs is widely accepted to be present in early infancy for purely functional reasons. The visual world is rich and complex and not all information is equally salient. By whatever means, pre-attuned or learned, infants must be able to select some information while paying little attention to other aspects of available stimuli. Such competence is particularly important for the process of categorization and, in his review, Bornstein (1984) states that “infants give evidence that, while still in the first months of life,
they categorize by perceptual equivalence in at least two modalities" (p. 321). These modalities are vision and audition, the same as those used in all the numerical discrimination tasks described above.

Therefore, what Wynn interprets as evidence of the representation of a semantically-rich system of numerical knowledge may simply be no more than the basic workings of the infant object representation system. Babies, it appears, are biased to generalize over perceptual inputs in at least two ways. They appear to prioritize spatiotemporal information when appropriate, thereby freeing them from reliance upon initially having to represent objects and events on the basis of complex and variable property information not necessary for the task at hand. They also carry out simple categorization, which enables similar entities to be treated as equivalent for certain purposes. There is nothing in those abilities that relates directly to the domain of number or the understanding of its conceptual structure.

**Physical Reasoning**

Wynn’s (1992, 1995) claim that infants understand ordinal and mathematical relations between small collections is founded upon their behavior in violation of expectation experiments. Infants indicated outcomes that did not accord with their expectations by looking longer at situations where dolls removed from behind a screen were still seen to be present and where dolls placed behind the screen were found to have disappeared. I contend that claiming that these behaviors indicate understanding of ordinal relations may be an overinterpretation of the data. However, what cannot be contested is that, at very least, infants must already have some basic form of the “object concept” in order to generate those reactions, arithmetical knowledge or no. The object concept requires that the child appreciates that an object that can no longer be seen nevertheless continues to exist. More importantly for the present discussion, therefore, it continues to be mentally represented. In the last ten years an ever-growing body of evidence has shown that “out of sight” is definitely not “out of mind” as far as the 5-month-old infant is concerned. Extensive studies by Baillargeon and her colleagues (Baillargeon, 1993, 1995) and Spelke and her colleagues (Spelke et al., 1992) among others have shown that infants just a few months old appreciate what have been referred to as a number of “principles” of the physics of simple real world objects.

Without such understanding about objects, infants would be unable to produce the surprise reactions observed in response to so-called “impossible” outcomes. If infants were not representing the persistence of a Mickey Mouse doll that they had watched being placed behind a screen, its apparent disappearance upon the lowering of the screen would not be noticed and would not induce a violation of expectation measurable by increased looking time. The real question is whether, along with basic memory abilities, an object file style of representation, and the ability to carry out cardinal discrimination of relative quantity, any further knowledge or understanding is required to explain infant behavior in these tasks.
Wynn (1995) claims that further arithmetical knowledge is required. Her argument is based on the outcome of one condition of our replication (Simon et al. 1995) of her original (Wynn, 1992) study. Reasoning that infants' behavior in Wynn's "addition/subtraction" study may simply have been due to the violation of expectations based not on arithmetic but physical knowledge, we added conditions where the number of items was held constant but the identity of the objects themselves was magically transformed. Thus, the infant might see the "Sesame Street" character Elmo on the stage, have the screen rise to occlude him and then see another Elmo placed behind the screen, only to witness Elmo and Ernie on stage when the screen was lowered. We reasoned that this would be recognized as an impossible event and so should invoke as much prolonged looking time as when an Elmo was joined by another Elmo but resulted in just a single Elmo. We even had evidence from habituation trials that a static Elmo could be distinguished from a static Ernie. However, no such result was found. The situation where Elmo and Elmo resulted in Elmo and Ernie produced no different response than when Elmo and Elmo resulted in Elmo and Elmo. Infants did not react to this resulting state in the same way they had done to other impossible outcomes.

Wynn states that this finding means that our infants "could not have been responding on the basis of expectations about the precise visual aspects of the display" (1995, p. 13) and this is entirely consistent with the view I have just presented. But neither does this fact indicate that numerical knowledge was being used. In fact the movement of dolls into view and then behind screens closely approximates aspects of the procedure used in the studies of Xu and Carey (1996) and Narter and Rosser (1996) which suggest that 9- and 10-month-old infants use spatiotemporal and not featural information as the primary representational basis for reasoning in these kinds of occlusion tasks. In other words, to paraphrase the understanding I believe that our 5-month-old infants had of the above situation, they were expecting that "a thing and a thing behind the screen should lead to a thing and a thing behind the screen," which it did! The point here is that, given the abstract coding of perceptually distinct items as units or things, our results and Wynn's are consistent with the claim that, in order to generate the behavior seen in all of these studies, infants require nothing more than the ability to carry out one to one correspondence between a remembered and visible collection of units, and the knowledge that things expected to persist should not disappear.

**CONCLUSIONS**

In this paper I have presented a new conceptualization of the origins of number knowledge in terms of what I have referred to as non-numerical competencies. These are primarily based on the individuation characteristics of the human attention system. Such competencies support only the most primitive numerical operations; "same/different" discriminations that are the functional equivalents of cardinal representations of quantity. Yet, these are all that are needed to explain the
behavior of infants in all the tasks published to date that have been designed to
evaluate the human infant's numerical abilities. This non-numerical view stands in
contrast to Wynn's (1995) claim that, at 5 months of age, infants' behavior on a
range of tasks provides evidence that they possess and can use an innately-specified
core set of numerical knowledge. This, Wynn claims, enables infants to enumerate
and reason numerically about small sets of static and dynamic objects and events.
While none of the behaviors described are disputed, I have presented the view that
it is not necessary to interpret these behaviors as evidence for a system of specifi-
cally-numerical knowledge, especially one so sophisticated as to encode ordinal
relations between quantities.

No competence beyond the ability to put representational tokens into one to one
correspondence is required by my non-numerical account. Discriminations based
on the proposed processes afford no reasoning about ordinal relationships and
restrict comparisons to the "same/not same" range. These are the only kinds of
judgments for which empirical studies have thus far produced direct evidence in
infants. The ability to make such judgments is also limited in these infants to very
small sets of entities. This suggests a subitizing like process, and not a procedure
like counting, is the basis of the infant competence that has been claimed to be
numerical. Eventually, what is "numerical" and what is "non-numerical" is a mat-
ter of philosophical debate. However, recent studies that have claimed to demon-
strate infant numerical abilities in ways other than those used by Wynn (1992) and
Simon et al. (1995) have still not required any competencies beyond those argued
for in my non-numerical account. I have already discussed Wynn's puppet-jump-
ing data in reference to her earlier (1995) paper. However, Wynn (1996) herself
adopts the term "individuation", that I have used here, as key to interpreting those
data as evidence for infants' "enumeration of actions." Canfield and Smith (1996)
also claim that five-month-old infants can execute sequential enumeration in a
visual expectation paradigm. Despite differences in methodology, the processing
required is quite similar to Wynn's puppet-jumping task. Canfield and Smith con-
clude their paper with a set of information processing requirements for their data
that are striking in their similarity to a subset of the ones I have presented here.
They say that infants "demonstrated their capacity (a) to treat pictures as units
whose sensory attributes are irrelevant to the prediction of their future position, (b)
to combine these units over time, and (c) to use the number of items accumulated
to predict the occurrence of [the target picture]" (p. 278). Requirement (a) is the
same as my abstraction requirement, (b) is the same as my memory requirement,
and (c) equates to my claim about limited representation and discrimination
because the set of units over which infants could successfully predict target picture
location was never larger than four. Canfield and Smith continue by saying that
"[the] nature of the relationship between the accumulated items and either a card-
dinal or ordinal representation of number cannot be determined given the design
of our study." This is consistent with my claim that no direct evidence for ordinal
concepts has yet been demonstrated.
The non-numerical view also implies a more realistic developmental trajectory for numerical development. From the initial foundation of same/different judgments based on objects’ existence, young children must construct a system of conceptual knowledge, indeed a language, for the meaning of quantities and the relationships between them. Along with this must occur the acquisition and mastery of procedures for forming quantitative representations beyond the range of the perceptual processes that support those initial discriminations. These include counting, along with a range of other enumeration and arithmetic strategies. Such processes provide access to the relationships between quantities such as “bigger” and “smaller” along with more complex relations such as “twice the size,” “the square of” etc. These pieces of information are presumably integrated with the functional equivalents of early cardinal concepts to extend the child’s conceptual representations of numbers. Such a view is consistent with two large bodies of literature. One chronicles the slow development both of counting procedures and number concepts throughout childhood (Fuson, 1988). The other shows how various procedures can be used to pressure the enumeration performance of children and adults. When this is done the result is always a dissociation between abilities that can be used to enumerate very small quantities in exactly the range infants have been shown to succeed, and those that can be employed for any other size of collection (Atkinson et al., 1976a; Chi & Klahr, 1975; Oyama, Kikuchi & Ichihara, 1981; Simon & Vaishnavi, 1996; Trick & Pylyshyn, 1993). This suggests that the simple perceptual discrimination-based competence I have claimed for infants is the primary means for making simple quantity-relevant judgments that both forms the foundation for more complex abilities during later development and continues to be used throughout the lifespan in the small number range.

Of course, in the end, this debate can only be resolved by careful experimentation and the collection of those data that can be used to distinguish between these two accounts. The early studies in this field have provoked enormous interest in answering the question of exactly what numerically-relevant skills human beings start out life with. Wynn (1995) has clearly articulated one way to interpret the data in order to answer that question, and in this paper I have presented a second interpretation. Hopefully there will be others, but more importantly, this debate should have the effect of stimulating sufficient research for us to converge on one account; the answer to the question of just what are the origins of numerical competence.

REFERENCES


